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Increasing Discrimination in Multi-criteria Analysis

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ABSTRACT

Multi criteria methods for supporting decision-making procedures are widely used in sustainability assessment. One of the most important steps in decision-making procedures is the evaluation of policy options or alternatives in order to find a hierarchy of option choices. Utility function value distributions are often constructed for the range of indicators for the options to be assessed. This distribution can be presented as an impact matrix stacked with indicator weights to reflect the relative importance of different indicators for the decision-maker. Solving rules are then introduced to integrate all individual indicator evaluations into a single integral utility estimation. These are often based on the averaging procedures, one of the simplest being arithmetic averaging. Whilst averaging rules are very attractive to decision makers due to their simplicity and logical transparency, using averaging as the first step of the decision-making procedures can significantly reduce the discrimination of the options, especially if there are counteractive individual indicator estimations.

This paper proposes a method to evaluate and overcome this loss of discrimination. The paper explains the basis of a discriminatory analysis approach to sustainability assessment, demonstrating its application through the use of illustrative data and describing its application to an existing case study where researchers had applied a number of multi criteria analysis tools. It was concluded that the discriminatory analysis provided a useful addition to the decision-makers toolbox as it provided a means of assessment of the validity of the application of the simple arithmetic averaging technique.

Key Words: Multi-criteria decision making tools; sustainability metrics and indicators

1. INTRODUCTION

Multicriteria methods for supporting of decision-making procedures have found an increasing recognition and widespread use. Such methods are capable of dealing with the multiple dimensions of evaluation problems (e.g. social, cultural, ecological, technological, institutional, etc.) and give due attention to conflicts among stakeholders involved. In general, the aim of these methods is to combine assessment methods with judgement methods and to offer a solid analytical basis for modern decision analysis (Nijkamp et al, 1990). One of the most important steps in decision-making procedures is the estimation of policy options or alternatives in order to find a hierarchy option choices. In this step the distribution $u_{i,j}$ of utility function values, versus policy options (i) and indicators (j) are frequently served as input for the evaluating processes. This distribution can be presented as an impact matrix stacked with indicator weights to reflect the relative importance of the indicators for the decision-makers.

A solving rule should then be introduced to integrate all individual indicator evaluations into a single integral utility estimation, which allows discrimination and ranking of the policy options. The solving rules are often based on the averaging procedures, one of the simplest being just arithmetic averaging:

$$U(PO_i) = \sum_j w_j u_{i,j} \,, \tag{1}$$

where $U(PO_i)$ is integral utility of policy option PO_i and $u_{i,j}$ is a utility function value for this policy option with respect to the indicator I_j.

The averaging solving rules are very attractive for the decision makers through their simplicity and logical transparency. On the other hand, averaging as the first step of the decision-making procedures can significantly reduce the discrimination of the policy options, especially if there are counteractive individual indicator estimations. This can create significant instability of the policy option ranking even under small perturbations of the impact matrix.

It is proposed that an appropriate way to overcome this difficulty is to initially evaluate the data set using a form of discrimination analysis thereby enabling an assessment to be made of the subsequent validity of data analysis using a simple arithmetic averaging technique, i.e. to change the usual order of multi criteria assessment of “estimating – discriminating – ranking” to the following one “discriminating – estimating – ranking”. Hereafter the new Discriminative Multi Attribute Ranking Technique (DISMART) is briefly described.

2. PROBLEM FORMULATION

Consider the weighted utility evaluation variance,

$$\mathrm{var}(I_j) = \frac{1}{n}\sum_i \left(b_{i,j} - \bar{b}_j\right)^2 , \qquad (2)$$

$$\bar{b}_j = \frac{1}{n}\sum_i b_{i,j} , \qquad (3)$$

where $b_{i,j} = w_j u_{i,j}$, to be suitable measure of indicator individual discriminative ability. The total discriminative ability D_{tot} of all indicators can be defined as a sum of individual discriminative abilities.

$$D_{tot} = \sum_{j=1}^{m} \mathrm{var}(I_j). \qquad (4)$$

To provide for the best discrimination of the policy options on the base of all indicator discriminative abilities the following policy option discriminator PCS_i can be introduced as linear transformation of indicator evaluations

$$PCS_i = \sum_{j=1}^{m} b_{i,j} a_j , \qquad (5)$$

Where a_j are unknown coefficients, which should be normalised

$$\sum_j a_j^2 = 1, \qquad (6)$$

and provide for the largest covariance of discriminator

$$Cov(PCS) = \frac{1}{n}\sum_i \left(PCS_i - \overline{PCS}\right)^2 = \max , \qquad (7)$$

$$\overline{PCS} = \frac{1}{n}\sum_i PCS_i \ . \qquad (8)$$

There should be found the set of coefficients $\{a_j\}$, which defines the discrimination scale where policy options are distributed in accordance to (5).

3. PROBLEM ANALYSIS

The problem (equations 5, 6 and 7) is exactly the same as it is considered in the frames of Principal Component Analysis (PCA). PCA is often used to reduce the dimension of a data set, replacing a large number of correlated variables with a smaller number of orthogonal variables that still contain most of the information in the original data set. PCA does not have an underlying statistical model, so it is just a simple mathematical technique.

According to PCA there are, in general, n sets of coefficients $\{a_j^{(1)}\}, \{a_j^{(2)}\}, \ldots, \{a_j^{(n)}\}$, which are eigenvectors of variance-covariance matrix

$$K_{j,k} = \frac{1}{n}\sum_{i=1}^{n}\left(b_{i,j} - \bar{b}_j\right)\left(b_{i,k} - \bar{b}_k\right), \tag{9}$$

to be correspondent to the eigenvalues $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n \geq 0$. These sets have variances equal to the correspondent eigenvalues; they are uncorrelated and usually called as principal components (PC's). The sum of eigenvalues equals to the total discriminative ability D_{tot} of indicators. Policy option discriminator

$$PCS_i^{(s)} = \sum_j b_{i,j} a_j^{(s)}, \tag{10}$$

correspondent to the s-th PC, is s-th principal component score (PCS) for the i-th policy option. The value

$$\Lambda_s = \frac{\lambda_s}{D_{tot}}. \tag{11}$$

is relative variance of the s-th principal component (PCRV).

It can be concluded that PC's provides for the multicriteria assessment by several (not more than number of indicators to be used) orthogonal discrimination scales where policy options are ordered independently by PCS's. It's worth to point out that PCA discrimination ordering can't be considered directly as ranking ordering because, inter alia, if set $\left\{a_j^{(s)}\right\}$ is s-th PC then set $\left\{-a_j^{(s)}\right\}$ is also PC correspondent to the same eigenvalue λ_s. That means the next stage of assessment is required to interpret PCA discrimination scales in order to find the ranking one. Some of modalities for the PCA interpretation in the frames of multicriteria assessment of policy options are illustrated by the following two case studies.

3. CASE SUDIES

3.1 Case Study 1. Model examples

Case Study 1 uses illustrative data to demonstrate the application of DISMART. Consider four extreme distributions of weighted utility function, reflecting different kinds of individual indicator interactions, which are presented in Tables 1 and 2. The results of arithmetic averaging (equation 1) and policy option ranking are presented in Table 3. It is clear from these figures that, except indicator agreement, policy options are poorly discriminated by the averaging thus ranking seems to be not reliable at least for the case of indicator disagreement.

Policy options	Indicators High Level of Agreement (HLA)						Indicators Moderate Level of Agreement (MLA)					
	I_1	I_2	I_3	I_4	I_5	I_6	I_1	I_2	I_3	I_4	I_5	I_6
PO_1	0.1	0.1	0.1	0.1	0.1	0.1	0.9	0.1	0.1	0.1	0.1	0.1
PO_2	0.3	0.3	0.3	0.3	0.3	0.3	0.7	0.3	0.3	0.3	0.3	0.3
PO_3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
PO_4	0.7	0.7	0.7	0.7	0.7	0.7	0.3	0.7	0.7	0.7	0.7	0.7
PO_5	0.9	0.9	0.9	0.9	0.9	0.9	0.1	0.9	0.9	0.9	0.9	0.9

Table 1: High and Moderate levels of agreement

Policy Options	Indicators Disagreement as conflict (DAC)						Indicators Disagreement as discordance (DAD)					
	I_1	I_2	I_3	I_4	I_5	I_6	I_1	I_2	I_3	I_4	I_5	I_6
PO_1	0.9	0.9	0.9	0.1	0.1	0.1	0.1	0.9	0.7	0.5	0.3	0.5
PO_2	0.7	0.7	0.7	0.3	0.3	0.3	0.3	0.1	0.9	0.7	0.5	0.5
PO_3	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.3	0.1	0.9	0.7	0.5
PO_4	0.3	0.3	0.3	0.7	0.7	0.7	0.7	0.5	0.3	0.1	0.9	0.5
PO_5	0.1	0.1	0.1	0.9	0.9	0.9	0.9	0.7	0.5	0.3	0.1	0.5

Table 2: Disagreement

Policy options	Indicator interactions							
	HLA		**MLA**		**DC**		**DD**	
	$U(PO)$	**Rank**	$U(PO)$	**Rank**	$U(PO)$	**Rank**	$U(PO)$	**Rank**
PO_1	0.1	5	0.23	5	0.5	Undefined	0.5	Undefined
PO_2	0.3	4	0.25	4	0.5	Undefined	0.5	Undefined
PO_3	0.5	3	0.5	3	0.5	Undefined	0.5	Undefined
PO_4	0.7	2	0.63	2	0.5	Undefined	0.5	Undefined
PO_5	0.9	1	0.77	1	0.5	Undefined	0.5	Undefined

Table 3: Policy option average utility estimations and ranking

Table 4 contains policy option discriminations of the first and second PCS's showing its significantly larger discrimination ability. Correspondent principal component relative variances are listed in the last row. According to these data the first PC, having PCRV equals to 1, totally describes policy option evaluation variability for the cases of indicator agreement and disagreement as conflict. In all cases policy options are well discriminated. If some kind of disagreement appears among indicator evaluations then values with opposite signs appear in the first PCS's. The stronger disagreement the more comparable extreme negative and positive First PCS's on absolute value becomes.

Policy options	**Indicator interactions**							
	HLA		**MLA**		**DC**		**DD**	
	First PCS's	Second PCS's	First PCS's	Second PCS's	First PCS's	Second PCS's	First PCS's	Second PCS's
PO_1	0.245	0	-0.163	0.525	-0.98	0.989	-0.096	-0.096
PO_2	0.735	0	0.327	0.525	-0.49	0.989	0.474	0.474
PO_3	1.225	0	0.816	0.525	0	0.989	0.388	0.388
PO_4	1.715	0	1.306	0.525	0.49	0.989	-0.234	-0.234
PO_5	2.205	0	1.796	0.525	0.98	0.989	-0.533	-0.533
PCRV	1	0	1	0	1	0	0.362	0.362

Table 4: Policy option PCS estimations

If there is high or moderate level of agreement existing in the indicator evaluations then the First PCRV takes the main part of the total discriminative ability and the first discrimination scale can be easily converted into the ranking scale by appropriate choice of direction "from bad to good". The obvious choice can be made, for instance, by comparing the policy option distributions along discrimination scales and policy option ranking by the average utility estimations (Table 4) that leads to the same ranking.

The disagreement conditions, when extreme negative and positive First PCS's have the same order of magnitude on absolute value, do not allow converting of the first discrimination scale into the ranking one. Under conditions of disagreement as conflict, when the First PCRV also takes the main part of the total discriminative ability, the first PCS's clearly signify the policy options, which are the subject of conflict, so the moderate decision could be proposed as compromise (for instance, policy option that corresponds to the median of the first PCS's distribution) or some kind of controlled dialogue could be initiated between stakeholders in order to soften detected conflict. If the median concept for compromise is acceptable then PO_3 will be taken as the suitable.

Under conditions of disagreement as discordance, when the total discriminative ability more or less equally distributed between PC's, one can conclude that several more or less independent conflicts exist between indicator estimations. The absence of concordance does not allow taking any reliable decision. The possible way out is to reconsider the set of indicators or to initiate dialogue between stakeholders.

3.2 Case Study 2 - Escape from the sewer system of domestic sanitary waste

The disposal of domestic sanitary solids using the WC as a “rubbish bin” is habitual in the UK and stems from the historical link associating health risks with human waste (hygiene). A multi-disciplinary comprehensive scientific study has been carried out in Scotland to consider the concept of sustainability for this issue and to address the three main component areas: economy, environment and society (Ashley et al., 2005). The following indicators (Table 5) and policy options (Table 6) were considered and three methodologies of multi criteria assessment were implemented, namely PROMETHEE, ELECTRE software packages and the Simple Multi-attribute Rating Technique (SMART).

Indicators	Code
Economic	
1. Capital costs	CC
2. Operational costs	OC
3. Maintenance cost	MC
4. Financial risk exposure	FRE
Social	
5. Acceptability (to Stakeholders)	AcSt
6. Perceived impact on the Environment	PIE
7. Participation and responsibility	PaRe
Environmental	
8. Sanitary waste escape	SWE
9. (Primary) Energy use	EnUs
10. Impact on air – CO_2	IoAC
11. Impact on air – NO_x	IoAN
12. Impact on air – SO_4	IoAS
13. CSO discharge	CSOD
Technical	
14. Sanitary waste transport	SWT
15. Risk of failure	RoF
16. Flexibility and Adaptability	FIAd

Table 5: Indicators

.Policy Options	Code
Install 6mm screens on overflows at waste-water treatment plants	In6S
“Think before you flush” (TBYF) campaigns	TDYF
Install flow storage	InSt
Retrofit storm water source control	RSSC
Sewer rehabilitation	SeRe
Retrofit outlet chokes on existing WCs and introduce these to new developments	ROC

Table 6: Policy options

The weightings shown in Table 7 and utility function values shown in Table 8 were used in the three methods of analysis to rank the policy options and to verify its stability in sensitivity testing. The results of this analysis from the case study report are shown in Table 9.

Code of indicator	Weight		
	Normal	Inverted	Equal
1. CC	10.5	2.5	6.25
2. OC	10.5	2.5	6.25
3. MC	10.5	2.5	6.25
4. FRE	3.5	7.5	6.25
5. AcSt	4.5	10.5	6.25
6. PIE	7.5	7	6.25
7. PaRe	3	17.5	6.25
8. SWE	8	2	6.25
9. EnUs	3.5	4	6.25
10. IoAC	3.5	4	6.25
11. IoAN	3.5	4	6.25
12. IoAS	3.5	4	6.25
13. CSOD	8	2	6.25
14. SWT	4	12.5	6.25
15. RoF	9	6	6.25
16. FIAd	7	10.5	6.25

Table 7: Variants of weighting

Indicator Code	Policy Option Code					
	In6S	TBYF	InSt	RSSC	SeRE	ROC
1. CC	45	80	49	95	65	70
2. OC	85	80	100	100	100	100
3. MC	35	100	80	100	100	75
4. FRE	70	100	0	100	100	0
5. AcSt	69	86	63	52	89	66
6. PIE	61	78	59	58	72	67
7. PaRe	0	100	0	100	0	100
8. SWE	95	75	60	40	50	90
9. EnUs	25	95	0	30	90	50
10. IoAC	80	95	70	85	90	95
11. IoAN	85	100	20	95	90	70
12. IoAS	60	95	10	85	90	80
13. CSOD	10	10	50	25	30	15
14. SWT	100	100	50	50	0	50
15. RoF	100	25	85	60	85	0
16. FIAd	60	60	0	100	0	0

Table 8: Utility function values

Policy Option	PROMETHEE			ELECTRE			SMART		
	Normal	Inverse	Equal	Normal	Inverse	Equal	Normal	Inverse	Equal
In6S	5	5	5	6	=4	5	4	3	4
TBYF	1	1	1	2	1	=1	1	1	1
InSt	6	6	6	5	6	6	6	6	6
RSSC	3	2	3	=3	3	=3	2	2	2
SeRe	2	3	2	1	2	=1	3	5	3
ROC	4	4	4	=3	4	=3	5	4	5

Table 9: Initial case study policy option ranking

Overall consideration of the results would indicate that the TBYF option dominates the top rankings, followed by SeRe (sewer rehabilitation) and then the RSSC (retrofit source control). The other three options are quite consistently placed in the final three positions. There would appear to be a reasonable degree of consistency between the outranking methods (PROMETHEE and ELECTRE) but there is less consistency between these methods and the SMART analysis. This is likely to be due to the influence of trade-offs in the smart analysis between positive and less positive scores for individual criteria within the analysis. Whilst the three techniques gave broadly similar results in terms of the policy option rankings, the rankings were found to be largely insensitive to changes in weightings giving rise to concerns about the discriminative abilities of the techniques.

Further assessment of the discriminative ability of the SMART analysis was undertaken in the initial case study using the normal weightings in Table 7 using Monte Carlo simulation. Deterministic estimates of the scores for criteria and there weightings were replaced by probability distributions. In the absences of sufficient data to allow the precise nature of the probability distributions to be determined, truncated normal distributions were used. Truncation was applied to prevent scores less than 0 or greater than 100 being used and the standards deviations of the distributions were selected to reflect estimates of the accuracy of the assessment of the scoring and weighting of the criteria. The results of the simulation are shown in Figure 1. The overlap between the pairs of distributions e.g. TBYF and RSSC may suggest that both policy options are similar in terms of their sustainability, but this may again illustrate a lack of discrimination in the SMART methodology arising from the trade-offs.

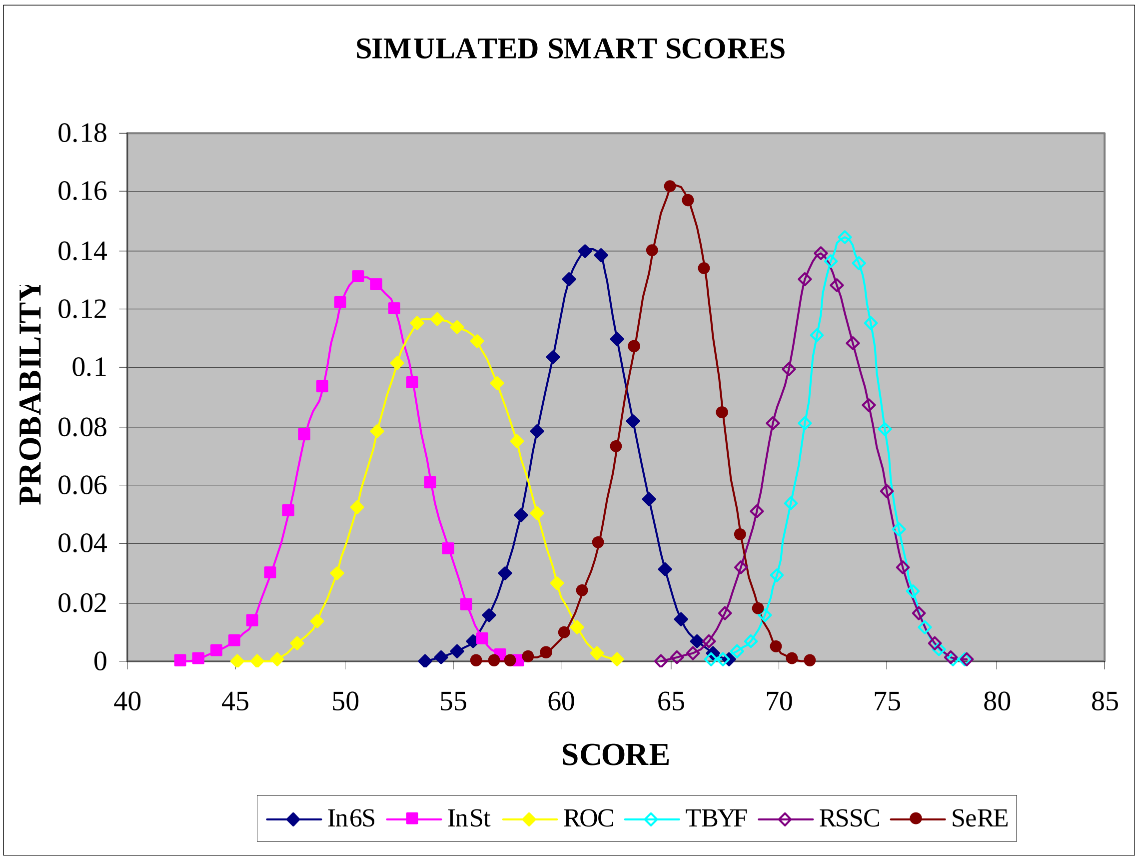

Figure 1: Simulated smart scores

To address this question, the data in tables 7 and 8 were analysed for this paper using the DISMART approach and the results are shown in Tables 10 and 11. The first PCS's are shown in Table 10. The first PCS orders the policy options on the maximum discrimination scale. Under the both inverted and equal weighting PCS's have the same sign (positive for the equal and negative for the inverted weighting). Then it can be taken that there are no disagreement indicator evaluations and discrimination scale can be transformed to the ranking scale by appropriate choice of direction for the rank descending: positive for the PCS of equal weight and negative for the PCS of inverted weight. The correspondent ranks are listed in Table 10.

If the normal weights are used then the first PCS's change sign that can be explained by the presence of moderate agreement because the absolute value of negative PCS is much smaller then largest positive. Then the discrimination scale can be also transformed to the ranking scale by the same choice of direction for the rank descending as it was done for the equal weighting.

Policy Option Code	First PCS		
	Normal weight	Inverted weight	Equal weight
TBYF	1044	-2107	1534
RSSC	933.9	-2042	1367
ROC	796.9	-1792	953.5
SeRe	405.1	-19.0	852.9
In6S	15.5	-357,6	648.9
InSt	-6.7	-32,0	75.8

Table 10: The first PCS under different weighting

Policy Option	DISMART		
	Normal	Inverse	Equal
TBYF	1	1	1
RSSC	2	2	2
ROC	3	3	3
SeRe	4	6	4
In6S	5	4	5
InSt	6	5	6

Table 11: DISMART policy option ranking

The correspondent distributions of eigenvalues are shown on Figures 2, 3 and 4. From these figures it is clear that first PC takes more than 40% of the total discriminative ability of all indicators which supports ranking based on the first principle component.

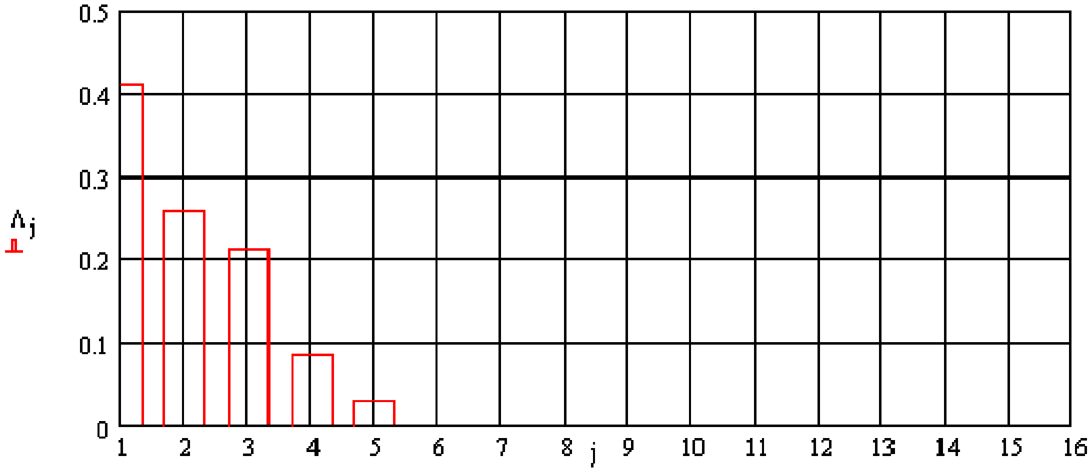

Figure 2: PCRV distribution under the normal weighting

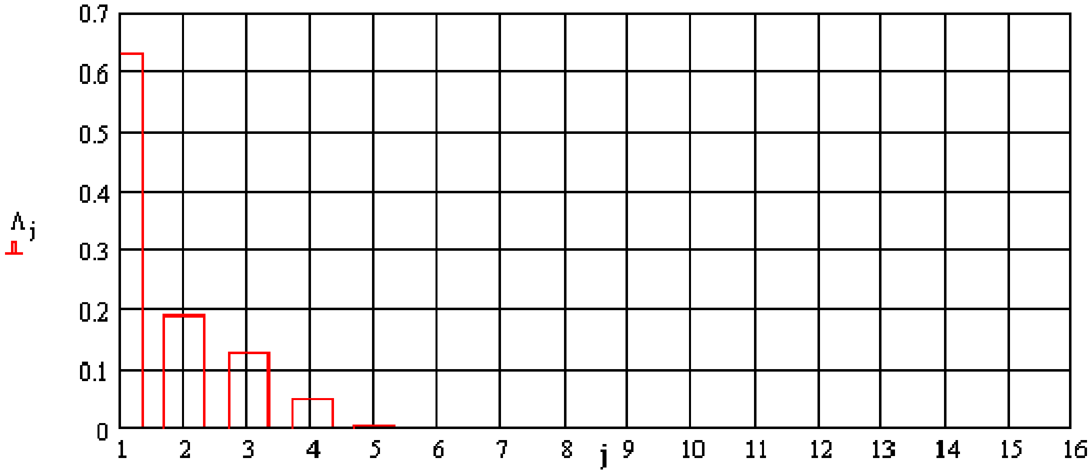

Figure 3: PCRV distribution under the inverted weighting

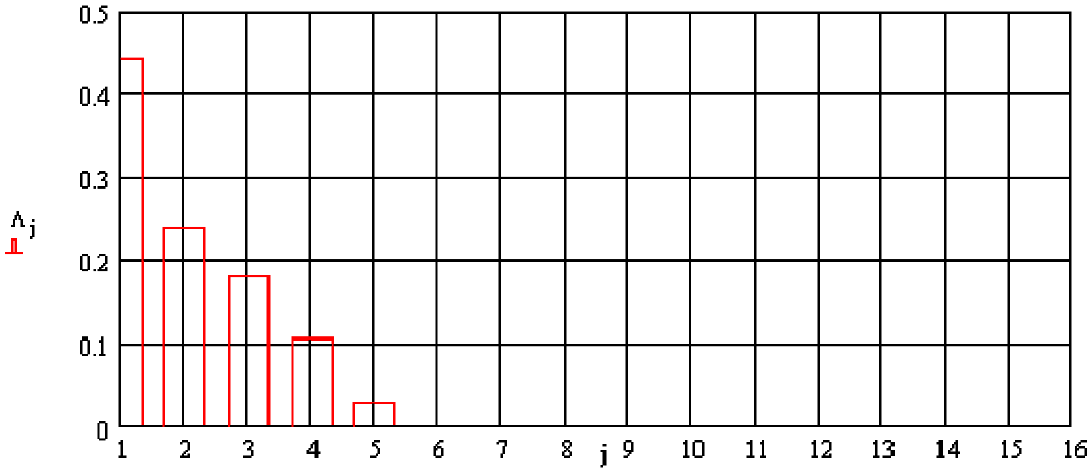

Figure 4: PCRV distribution under the equal weighting

4. CONCLUSIONS

The DISMART analysis demonstrates adequate discrimination between the policy options in the initial case study analysis, which provides evidence that the ranking obtained from the SMART analysis were appropriate. Furthermore, the DISMART analysis demonstrates that the sensitivity testing in the initial case study discriminated adequately options when the weightings were changed which indicates that the lack of sensitivity in the ranking of the options is a true reflection of their relative sustainability and not a result of known methodological limitations of SMART analysis.

Consequently DISMART provides a useful addition to the decision-makers toolkit. Simple such as SMART are very attractive to decision makers due to their simplicity and logical transparency, and have been shown in Case Study 2 and by other researchers to provide similar results to more complex mathematical methods such as ELECTRE. However, it is known that averaging can significantly reduce the discrimination of the options, especially if there are counteractive individual indicator estimations. The use of DISMART prior to a Smart analysis will enable the decision maker to assess the influence of counteractive individual indicators within a data set thereby providing guidance on the appropriateness of a simple arithmetic averaging technique.

The principle of maximum discrimination of policy options before the multi criteria estimations leads to more information regarding the subsequent ranking of policy options. DISMART enables:

- Maximum discrimination of policy before assessment therefore taking into account all “viewpoints” of individual evaluations (indicators).
- Detection and measurement of the level of agreement.
- The introduction of discrimination scales for the scoring and ranking of policy options when agreement exists between indicator estimations.
- Detection of situations when disagreement as conflict exists and to propose the compromised decisions.
- Detection of situations when disagreement as discordance exists.

Future research will be carried out comparing the sensitivity analysis of DISMART results to other multi-criteria ranking approaches.

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